Chapter 5
Intermittent Heating Calculations

13.8 INTERMITTENT HEATING CALCULATIONS

Péclet was probably the first to discuss intermittent operation of heating systems in a quantitative manner.[46] He appreciated the fact of heat storage in the structure, and made crude attempts to compare the quantity of heat to be replaced after cooling with the quantity of heat lost by steady transmission. He knew of Fourier's work, and applied it to the penetration of sinusoidal temperature waves into solids, but he did not go on to use it for intermittency calculations. In spite of his attempts Péclet failed to give the designer any guidance as to sizing or energy use. It was, he said, impossible to calculate, even approximately, the amount of fuel used when heating is intermittent, on account of the large quantity of heat absorbed by the walls.

In order to calculate the time lag when the outdoor temperature changed, Péclet estimated the mean wall temperatures in the steady states corresponding to the two outdoor temperatures, and using the thermal capacity of the wall, worked out the change in the heat content of the wall. Taking a uniform rate of heat loss equal to the average of the two steady values, he was then able to determine the length of time over which the change of heat content takes place.

Box, who was aware of Péclet's work, made some attempt to calculate heat flows during intermittent operation.[12]

In his example (a school) the steady heat loss through the walls and windows was calculated to be 8983 Btu/h, the ventilation loss 11938 Btu/h, giving a total of 20921 Btu/h. The metabolic gain from 100 children was set at 19100 Btu/h, and thus almost enough to maintain the steady temperature rise of 30°F. The quantity of heat needed to raise the wall temperature from an initial 30°F to its steady mean temperature of 41°F is 367420 Btu. Box calculates that each square foot of wall receives from the stove pipe at 800°F an average of 73.1 Btu/ft^{2}h during the heating-up period by radiation and convection, and taking account of the conduction losses, a total of 92815 Btu/h is stored in the walls. Hence 367420/92815 = 4 h, nearly, is required to heat the school from cold. The cooling time is 367420/4492 = 82 h (4492 Btu/h being the *average* loss during cooling).

Box observes that:

> "This agrees with our experience that in a crowded room artificial heat is not necessary, except to warm the walls etc. beforehand, and in most cases the proportions of the heating apparatus must be fixed with special reference to the preliminary heating of the building, which we have done in this case."

He goes on to point out that because of the much smaller radiant component from a low temperature source, the heat entering the walls during preheating is less, and the warming-up period correspondingly longer.

In his calculations, he assumed that surface resistance was the only controlling factor, and that the physical properties of the wall (other than the specific heat) had no influence on periodic or transient heat flow into the wall. Yet although his assumptions are suspect, he was able to show that for at least one building (Église St. Roch), theory and experiment agreed in suggesting a preheating period of 8 days. His calculations are noteworthy, too, in that he took account of the thermal capacity of the heating system itself.

Box goes on to demonstrate for buildings which are used infrequently, continuous heating may use only a little more fuel than intermittent heating. As an example, he takes a church which is used one day a week. With intermittent operation, the weekly fuel use would be 678 lb. On the other hand, continuous firing throughout the week to maintain the steady temperature would consume 940 lb. Box thought that this could be reduced in practice, perhaps to 780 lb, owing to the greater efficiency of regular and slow firing, and "the church would always be ready for week-night or occasional services, and the convenience of this mode of heating are so great that it should become general".

German engineers (including Rietschel[52]) were soon making use of an empirical formula which involved the preheating and cooling times, namely:

$$\text{Addition to steady capacity} = \frac{0.0625(N-1)W}{Z}$$

where W = steady losses by conduction through fabric
N = number of hours heating off
Z = preheating time, hours.

Typically this formula gave the following additions:

	$N = 8$	$N = 12$
$Z = 2$	0.22	0.34
3	0.15	0.23
4	0.11	0.17

Hoffman and Raber seem to have been influenced by the German engineers: the intermittent heating allowances they quoted in 1913 were of German origin.

Rietschel deprecated the addition of more than one-third to the steady-state load, as to do so would increase running costs considerably. He recommended instead a longer preheating time or continuous operation. These formulae, as with his steady state calculations, do not include the ventilation loss: this was handled separately.

For very large spaces, it was deemed unnecessary to try to establish a steady state. Instead, the air within was to be warmed rapidly by a large heat input; and in this way, there is hardly any penetration into the walls, and thus no heat loss to outside through them. Only the windows allowed direct heat flow to outside.

Rietschel proposed formulae consisting of a term giving the average heat loss through the windows and a second term representing the heat storage in the fabric. It is noteworthy that in it he used the area of *all* the bounding surfaces, foreshadowing the influence function of Nessi-Nisolle and the absorbance of Smith.

The underlying philosophy of preheating was well understood by Debesson (1908), who wrote:(18)

> "There is for each building a certain coefficient, which M. Ser calls the coefficient of thermal inertia, which cannot be calculated but only found from experience, but which can define the power of a heat source necessary to establish and maintain the temperature of the building."

He adds that the calculation of continuous heating is relatively easy; but the sizing of plant for intermittent operation is impossible and can only be done by approximation. The number of calories to be provided depends on the building, the thickness of the walls, the area of windows and the air change. He continues:

> "Those who have the courage to face this difficult problem make a normal calculation of heat loss, and then add a variable percentage derived from their experience, and based on a comparison with a similar building. They arrive by chance at a figure which may correspond with demand, and they trust to their lucky stars that the result will be satisfactory, or that the judge will be lenient."

He gave a series of diagrams which showed quantitatively how the input power, the preheat time and the steady losses were related; and he observed that as the steady state is approached, the necessary input falls towards the steady value.

Barker (1912) very well knew the time lag involved in starting up a heating system.(7) He estimated the additional power, over and above the steady loss, by calculating the quantity of heat required to raise the wall temperature from cold to its steady value. He realised that this was only an approximation.

"If therefore heat is supplied only during the day, the amount supplied must be sufficient to provide for the loss that takes place at night as well as during the day. The orthodox method of calculating heat losses either leaves this essential fact out of sight or attempts in a somewhat feeble manner to provide for it by the addition of an arbitrary percentage. This addition is a proof that so far as the coefficients employed are found to be satisfactory in practice they are essentially based on the results of practical experience, and not on their correctness from the absolute or scientific standpoint... . It is not a question of scientific accuracy, but of practical adequacy."

In spite of this castigation, Barker goes on to advocate an approximate method of calculating heat losses (neglecting *U*-values) and adding 15% for rooms heated only during the day, or 35% for spaces which are infrequently warmed.

Little further progress, either theoretical or empirical, was made until the classic work of Nessi and Nisolle (1947).[44] In it they developed a complete theory based on the assumption of a sudden rise of indoor temperature (a so-called unit step), and defined two parameters which they termed "fonctions d'influence" — one, $g(t)$ referring to the effects of a change of inside temperature, and the other $e(t)$ the effects of a change of outdoor temperature (Fig. 13.4).

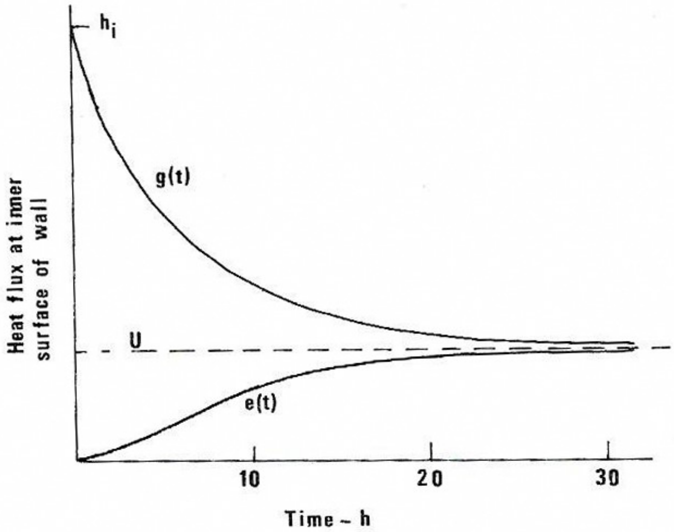

Fig. 13.4. Influence functions.

Unfortunately, the integrals involved were difficult to evaluate. Nessi and Nisolle devised a mechanical integrator with graphical output to perform the calculation of $g(t)$ and $e(t)$, and they presented the results in tabular form. Even then, the completion of the calculations for specific cases involved the summation of a series of values of $g(t)$ for the separate elements of the structure, and a second summation if the temperature variation was different from a unit step.

It is feared that Nessi and Nisolle's work was little applied in practice, even in France, though Cadiergues *et al.*[13] made some attempt to simplify its application (*ca.* 1952).

A. F. Dufton, working at BRS, made what are possibly the first scientific experimental studies of intermittent heating. He was concerned to show the value of low-thermal capacity linings (such as panelling or carpets) in intermittent heating. He developed a very simple formula to calculate the preheating time for a homogeneous wall.[21] This was extended by Griffith and Horton to apply to two-layer walls, and provided the theoretical basis for low-thermal capacity linings.[26]

The "absorbance" method by Elmer Smith (1941) was based on sound theory, but was developed to a practical design tool. It was in essence similar to that of Nessi-Nisolle, and suffered from the same disadvantages. Smith recognised the significance of internal furnishings, etc. as contributing to heat storage. Earlier a number of graphical methods, based on the Schmidt technique, were evolved for infrequently used buildings such as churches, and these methods were used by the gas industry in the United Kingdom (*ca.* 1945) for estimating the necessary heating power.

A little known work by Shklover[56] developed the matrix analysis of sinusoidal temperature waves through single and compound walls. Shklover's work is notable in that it introduces the admittance, though it is not so called. In 1949, Dusinberre's textbook on "Numerical Analysis of Heat Flow" was published.

Application of matrix analysis, and of the response factors proposed in 1956 by Brisken and Reque and developed by Stephenson[58] in Canada had to await the evolution of the computer.

Others, notably Stoef,[59] E. Harrison[28] and Barcs[5] have also made contributions both to system design and building design. Marmet made use of the electrical analogy of heat flow, and produced charts by which the impedance of a structure could be found. His theory, unlike the admittance procedure, included both amplitude and phase of the heat flow. He did not, however, apply his theory to the practical problem of designing for intermittent operation.

Krischer studied the pull-down time for refrigerated stores (the converse problem), and Bruckmayer (1951) sought to establish a simple parameter which would define heating or cooling rates. He chose a time constant Q/U which is valid only for the effects of internal changes, and which moreover is difficult to calculate precisely, because of the effects of ground storage on the value of Q. Nevertheless, it afforded some useful, if temporary, means of comparing simple and complex structures.

Experimental studies of intermittent heating were carried out by HVRA in the 1960's, and these led to a new empirical design tool (Fig. 13.5), which for the first time introduced the thermal inertia of the heating system.[11]

From its introduction, off-peak electric floor-warming led to conflict with the traditional heating industry. It seemed to the latter that the electrical designers were providing systems which were manifestly too small to give the required temperatures. This is perhaps surprising, for in 1934, Smith[57] (concerned with water-storage systems) insisted that the overall design must ensure a true heat balance, i.e. the heat put in during the off-peak charging period must equal the 24-h usage. For the design of systems to be operated intermittently, Smith, like Box, considers the heat required to raise the mean wall temperature to the desired steady-state value. He shows that for a 12-in masonry wall, this may be 32 Btu/ft^{2}h for a 3-h preheating period, as compared with the U-value of 6 Btu/ft^{2}h. He recommended the use of Rietschel's intermittent heating formula; he preferred night set-back to complete shut-off.

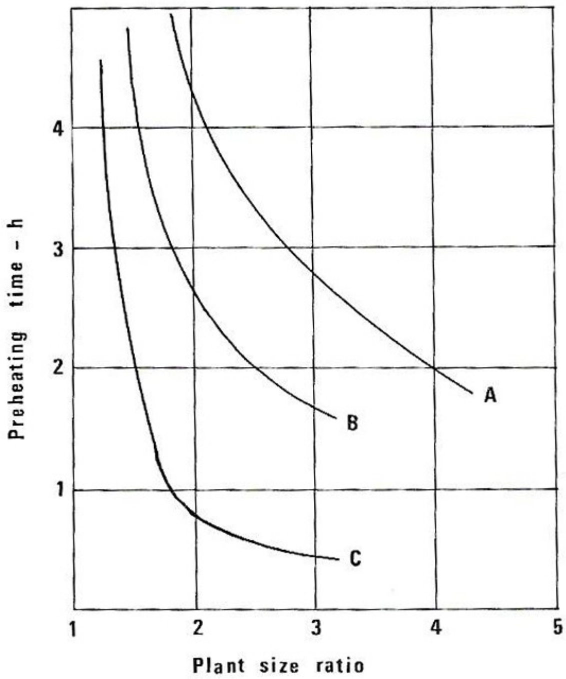

Fig. 13.5. Preheating time.

The conflict is well illustrated in a paper by H. Bruce to IHVE, and the discussion on it is sharp and pointed, if not actually acrimonious. But the investigations to which it gave rise proved valuable in the development of methods of estimating energy consumption in any kind of system. A scientific study of intermittent heating and of off-peak floor-warming was carried out by HVRA between 1958 and 1960, and this brought to light the major reasons for the apparent discrepancies – the importance of miscellaneous heat gains from other sources, the significance of the thermal capacity of the building, and perhaps excessive allowance for ventilation. All these factors tend to reduce the energy use in off-peak systems below that which the conventional theory led the engineer to expect.

At about the same time, Danter and his colleagues at BRS were developing the admittance procedure, apparently in ignorance of the work of Shklover, but the use of computer evaluation of the matrix, and the restriction to a 24-hour period, made tabulation and the application far simpler. The HVRA empirical approach, and its estimates of input power, and energy saving, were confirmed by Danter's theory.

The great advance represented by the admittance procedure is that it is applicable not only to intermittent heating, but also to problems of summer cooling. Billington and Harrington-Lynn both applied the concept to the estimation of preheating times and the corresponding energy consumptions. The final advance was the introduction by Billington of a dimensionless parameter relating admittance and transmittance to describe the thermal weight of a structure (1974-5).

In very recent years, following the energy crisis, intermittent operation has become the norm, for commerce and industry as well as for the householder. This being so, it has become essential to design for this mode of use, rather than the steady state as had previously been the case. Even more important, perhaps, is the current realisation that it is necessary to design the *thermal* properties of the structure also.

APPENDIX 2.A

INTERMITTENT HEATING OF ONE- AND TWO-LAYER WALLS

(a) *Single-layer wall* [35]

A homogeneous wall, of thickness d, and conductivity k, has one face maintained at a constant temperature, assumed zero. The other face is heated at a constant rate $Q=(p\theta k/d)$. θ is thus the steady temperature of the heated face which would be maintained by a heat input $\theta k/d=Q/p$.

The complete solution for the temperature at any point in the wall, distant x from the heated surface, is:

$$\theta_{x,t}=p\theta\left\{1-\frac{x}{d}-\frac{8}{\pi^2}\sum_{n=0}^{\infty}(2n+1)^{-2}\,.\,e^{\frac{-(2n+1)^2\pi^2h^2t}{4d^2}}\,.\,\cos(2n+1)\pi x/d\right\};$$

and the temperature at the surface $x=0$ is:

$$\theta_{0,t}=p\theta\left\{1-\frac{8}{\pi^2}\sum_{n=0}^{\infty}(2n+1)^{-2}\,.\,e^{\frac{-(2n+1)^2\pi^2h^2t}{4d^2}}\right\} \qquad (2.9)$$

Mathematical solution by BILLINGTON
From THERMAL PROPERTIES OF BUILDINGS 1952

The surface attains the temperature θ when

$$\frac{8p}{\pi^2}\sum(2n+1)^{-2}.\,e^{-\frac{(2n+1)^2\pi^2h^2t}{4d^2}}=p-1 \qquad (2.10)$$

i.e. approximately when

$$t=\frac{4d^2}{5h^2p^2} \quad \text{if} \quad p>1{\cdot}5 \qquad (2.11)$$

This formula may be expressed in an alternative form. We have

$$Q=\frac{p\theta k}{d}, \quad \text{whence} \quad p=\frac{Qd}{\theta k};$$

so that

$$t=\frac{4d^2}{5h^2p^2}=\frac{4\theta^2k^2}{5h^2Q^2}$$

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i.e.

$$\theta=\frac{Q}{2}\sqrt{\frac{5t}{kc\rho}}\ * \qquad (2.12)$$

(b) *Two-layer wall*[49]

The front component has a thickness α; the back component is semi-infinite. The front face is heated at a constant rate Q.

The temperature at a distance x from the surface of the front component is given by:

$$\theta_1=\frac{Q}{k_1}\left\{\left[2\sqrt{\frac{h_1{}^2t}{\pi}}\,.\,e^{-(x^2/4h_1{}^2t)}-x\left(1-erf\frac{x}{2\sqrt{(h_1{}^2t)}}\right)\right]\right.$$

$$-\frac{1}{\gamma}\sum_0^\infty\left(-\frac{1}{\gamma}\right)^n\left[2\sqrt{\frac{h_1{}^2t}{\pi}}\,.\left(e^{-\frac{(x+2\alpha(n+1))^2}{4h_1{}^2t}}+e^{-\frac{(x-2\alpha(n+1))^2}{4h_1{}^2t}}\right)\right.$$

$$-(x+2\alpha(n+1))\left\{1-erf\left(\frac{-x-2\alpha(n+1)}{2\sqrt{(h_1{}^2t)}}\right)\right\}$$

$$\left.\left.+(x-2\alpha(n+1))\left\{1-erf\left(\frac{-x+2\alpha(n+1)}{2\sqrt{(h_1{}^2t)}}\right)\right\}\right]\right\}$$

where $\quad 0<x<\alpha;\ \gamma=\dfrac{k_1c_1\rho_1+k_2c_2\rho_2}{k_2c_2\rho_2-k_1c_1\rho_1}.$

* The temperature rise in a semi-infinite solid is

$\theta_{x,t}=2Q\sqrt{\dfrac{t}{\pi kc\rho}}\,.\,e^{-(x^2/4h^2t)}-\left(\dfrac{2x}{k}\right)\displaystyle\int_{x/\sqrt{(4h^2t)}}^{\infty}e^{-u^2}du;$ so that $\theta_{0,t}=2Q\sqrt{\dfrac{t}{\pi kc\rho}}$

Note that $2/\sqrt{\pi}=1{\cdot}13$; whereas $\sqrt{5}/2=1{\cdot}12$.

The temperature in the backing material is given by:

$$\theta_2=\frac{2Q\lambda h_1}{\gamma}\sum_0^\infty\left(-\frac{1}{\gamma}\right)^n\left\{2\sqrt{\frac{h_1{}^2t}{\pi}}\,.\,e^{\frac{-\left\{x-\alpha\left(1-\frac{h_2}{h_1}.\overline{2n+1}\right)\right\}^2}{4h_2{}^2t}}\right.$$

$$\left.-\left[x-\alpha\left\{1-\frac{h_2}{h_1}(2n+1)\right\}\right]\left[1-erf\left(\frac{x-\alpha\left(1-\frac{h_2}{h_1}.\overline{2n+1}\right)}{2\sqrt{(h_2{}^2t)}}\right)\right]\right\}$$

where $\lambda=(k_2h_1-k_1h_2)^{-1}$

The surface temperature (at $x=0$) is thus:

$$\theta_0=\frac{Q_1}{k_1}\left[2h_1\sqrt{\frac{t}{\pi}}-\frac{4}{\gamma}\sum_0^\infty\left(-\frac{1}{\gamma}\right)^n\left\{h_1\sqrt{\frac{t}{\pi}}\,.\,e^{-\frac{\alpha^2(n+1)^2}{h_1{}^2t}}\right.\right.$$

$$\left.\left.-\alpha(n+1)\left[1-erf\frac{\alpha(n+1)}{h_1\sqrt{t}}\right]\right\}\right] \quad (2.13)$$

(c) *Multi-layer walls*

Solutions have been given by various authors: they are, however, complex, and the original papers should be consulted.[96]