Chapter 13 Part 2
HEATING AND VENTILATING DESIGN

13.6 WILD HEAT

Heating engineers have, as a class, been concerned to avoid the risk of underheating, and as a result many systems have been considerably oversized. The loads are calculated for an empty, dark building, whereas in fact, buildings are occupied and lit. The heat dissipation from people, lights and machinery, together with that from sunshine through the windows, can be quite considerable — it is indeed the *raison-d'etre* of the degree-day base.

The value of this wild heat is reducing the demand even in design weather was not often appreciated. However, Hoffman and Raber (US, 1913) said that heat gains from lights, and metabolic heat, could be credited to the heating system.[29] Fowler[24] on the basis of observed energy consumptions in similar rooms, drew attention to this contribution, but he did not go so far as to advocate any reduction in the design load. It remained for the electrical industry to capitalise on these gains, albeit unwittingly, in their design techniques for storage heating. HVRA[1] through measurements in a number of buildings, demonstrated that maximum loads were in fact reduced by the miscellaneous gains, and that it was legitimate to deduct them from computed steady-state structural and ventilation losses in order to arrive at a suitable plant size (1961).

13.7 DEGREE-DAYS

The quantity of fuel used in a heating season to maintain a room or building at a given temperature is expected to depend on the difference between the inside and outdoor temperatures and upon the duration of this difference. The concept of the "degree-day" to quantify the duration and severity of a climate originated with Sir Richard Strachey in 1878, who used it for agricultural purposes. The first application of the degree-day to heating problems was due to the American Gas Association.* They found, from a survey of a large number of buildings, that no fuel was consumed when the daily mean outdoor temperature rose above 65°F (18°C) (for an internal temperature of 70°F (21°C)), and that the fuel consumption on any one day was proportional to the difference between 18°C and the 24-h mean outdoor temperature. If this difference is, for example, 10°K, then there are 10 degree-days for that day. The annual total is the sum of the number of degree-days for each day of the year, and the annual fuel usage is proportional to the annual degree-day total.

The temperature above which no fuel is needed for heating — in this case 18°C — is termed the "base" of the degree-day; the difference between this base and the maintained indoor temperature arises from miscellaneous sources of heat within the building — lights, people and solar gains. Once the base is given, the degree-day total is easily computed from meteorological records for the locality.

Dufton[20] published a map of degree-days for the British Isles in 1934, using a base of 60°F (15°C) to correspond with the British indoor temperature of 65° (18°C). He thus continued the American assumption of a 5°F difference between base and indoor temperatures. The idea of degree-days also gained a foothold in Europe, though the basis was rather different. There, degree-days were counted only for the heating season, and this too was variously defined as the period of the year when the daily mean fell below the indoor design temperature or below some other (lower) temperature designated the heating limit.

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*In 1915, Eugene D. Milener, an engineer with the gas utility company in Baltimore, found that gas consumption of heating plants in that city varied with the number of degrees difference between 64°F (17.8°C) and the outside temperature. Later studies indicated an improved relationship if 65°F (18.3°C) was chosen.

Grierson (1940) observed that not all buildings in Britain were warmed to 18°C, and suggested that degree-day totals were needed for a wide range of indoor temperatures.[25] He calculated figures for a 5-year period at Kew, stating only the "base" and not specifying a corresponding indoor temperature.

Knight and Cornell (1958) queried the propriety of applying conventional degree-days to buildings which were not continuously heated to a constant temperature. They proposed the use of a special degree-day for this purpose.[36] In a later paper (1966) Billington rejected the Knight-Cornell approach, preferring to retain the degree-day as a purely climatic statistic.[10] He identified the difference between the indoor temperature and the base temperature as the temperature rise caused by the miscellaneous heat gains to the building. It became clear from work at HVRA that the miscellaneous gains and their effects differed considerably from building to building, and hence that the use of a single base was no longer valid. Accordingly, Billington suggested a range of base temperatures appropriate to different structures and occupancy, while retaining the original dependence of the degree-day on outdoor temperature alone. At the time it appeared that the need for different base temperatures had also been recognised in France, but no basis for the choice of these bases had been published.

Somewhat earlier (1962) it had been proposed by Arnsted in Denmark to modify the degree-day to take account not only of temperature but also of the incidence of wind and sun, both of which affect the heat loss from a building. His corrections are to be applied only to degree-day totals for short periods such as a week or two. Arnsted states that the conventional total is adequate for the estimation of the fuel usage over long periods such as a complete heating season.

Attempts have also been made to evaluate summer (cooling) degree-days. These have so far failed because a large part of an air-conditioning load is latent heat and solar radiation through windows, and thus not directly related to the outdoor temperature.

A concept used to estimate the operating cost of refrigeration plant is that of "equivalent full-load operating hours". The same concept has been used in heating work,[10] both in Britain (chiefly at first by the electricity industry) and in Germany (often for district heating consumers). It is of course closely related to the annual degree-day total.

13.8 INTERMITTENT HEATING CALCULATIONS

Péclet was probably the first to discuss intermittent operation of heating systems in a quantitative manner.[46] He appreciated the fact of heat storage in the structure, and made crude attempts to compare the quantity of heat to be replaced after cooling with the quantity of heat lost by steady transmission. He knew of Fourier's work, and applied it to the penetration of sinusoidal temperature waves into solids, but he did not go on to use it for intermittency calculations. In spite of his attempts Péclet failed to give the designer any guidance as to sizing or energy use. It was, he said, impossible to calculate, even approximately, the amount of fuel used when heating is intermittent, on account of the large quantity of heat absorbed by the walls.

In order to calculate the time lag when the outdoor temperature changed, Péclet estimated the mean wall temperatures in the steady states corresponding to the two outdoor temperatures, and using the thermal capacity of the wall, worked out the change in the heat content of the wall. Taking a uniform rate of heat loss equal to the average of the two steady values, he was then able to determine the length of time over which the change of heat content takes place.

Box, who was aware of Péclet's work, made some attempt to calculate heat flows during intermittent operation.[12]

In his example (a school) the steady heat loss through the walls and windows was calculated to be 8983 Btu/h, the ventilation loss 11938 Btu/h, giving a total of 20921 Btu/h. The metabolic gain from 100 children was set at 19100 Btu/h, and thus almost enough to maintain the steady temperature rise of 30°F. The quantity of heat needed to raise the wall temperature from an initial 30°F to its steady mean temperature of 41°F is 367420 Btu. Box calculates that each square foot of wall receives from the stove pipe at 800°F an average of 73.1 Btu/ft^{2}h during the heating-up period by radiation and convection, and taking account of the conduction losses, a total of 92815 Btu/h is stored in the walls. Hence 367420/92815 = 4 h, nearly, is required to heat the school from cold. The cooling time is 367420/4492 = 82 h (4492 Btu/h being the *average* loss during cooling).

Box observes that:

> "This agrees with our experience that in a crowded room artificial heat is not necessary, except to warm the walls etc. beforehand, and in most cases the proportions of the heating apparatus must be fixed with special reference to the preliminary heating of the building, which we have done in this case."

He goes on to point out that because of the much smaller radiant component from a low temperature source, the heat entering the walls during preheating is less, and the warming-up period correspondingly longer.

In his calculations, he assumed that surface resistance was the only controlling factor, and that the physical properties of the wall (other than the specific heat) had no influence on periodic or transient heat flow into the wall. Yet although his assumptions are suspect, he was able to show that for at least one building (Église St. Roch), theory and experiment agreed in suggesting a preheating period of 8 days. His calculations are noteworthy, too, in that he took account of the thermal capacity of the heating system itself.

Box goes on to demonstrate for buildings which are used infrequently, continuous heating may use only a little more fuel than intermittent heating. As an example, he takes a church which is used one day a week. With intermittent operation, the weekly fuel use would be 678 lb. On the other hand, continuous firing throughout the week to maintain the steady temperature would consume 940 lb. Box thought that this could be reduced in practice, perhaps to 780 lb, owing to the greater efficiency of regular and slow firing, and "the church would always be ready for week-night or occasional services, and the convenience of this mode of heating are so great that it should become general".

German engineers (including Rietschel[52]) were soon making use of an empirical formula which involved the preheating and cooling times, namely:

$$\text{Addition to steady capacity} = \frac{0.0625(N-1)W}{Z}$$

where W = steady losses by conduction through fabric
N = number of hours heating off
Z = preheating time, hours.

Typically this formula gave the following additions:

	$N = 8$	$N = 12$
$Z = 2$	0.22	0.34
3	0.15	0.23
4	0.11	0.17

Hoffman and Raber seem to have been influenced by the German engineers: the intermittent heating allowances they quoted in 1913 were of German origin.

Rietschel deprecated the addition of more than one-third to the steady-state load, as to do so would increase running costs considerably. He recommended instead a longer preheating time or continuous operation. These formulae, as with his steady state calculations, do not include the ventilation loss: this was handled separately.

For very large spaces, it was deemed unnecessary to try to establish a steady state. Instead, the air within was to be warmed rapidly by a large heat input; and in this way, there is hardly any penetration into the walls, and thus no heat loss to outside through them. Only the windows allowed direct heat flow to outside.

Rietschel proposed formulae consisting of a term giving the average heat loss through the windows and a second term representing the heat storage in the fabric. It is noteworthy that in it he used the area of *all* the bounding surfaces, foreshadowing the influence function of Nessi-Nisolle and the absorbance of Smith.

The underlying philosophy of preheating was well understood by Debesson (1908), who wrote:[(18)]

> "There is for each building a certain coefficient, which M. Ser calls the coefficient of thermal inertia, which cannot be calculated but only found from experience, but which can define the power of a heat source necessary to establish and maintain the temperature of the building."

He adds that the calculation of continuous heating is relatively easy; but the sizing of plant for intermittent operation is impossible and can only be done by approximation. The number of calories to be provided depends on the building, the thickness of the walls, the area of windows and the air change. He continues:

> "Those who have the courage to face this difficult problem make a normal calculation of heat loss, and then add a variable percentage derived from their experience, and based on a comparison with a similar building. They arrive by chance at a figure which may correspond with demand, and they trust to their lucky stars that the result will be satisfactory, or that the judge will be lenient."

He gave a series of diagrams which showed quantitatively how the input power, the preheat time and the steady losses were related; and he observed that as the steady state is approached, the necessary input falls towards the steady value.

Barker (1912) very well knew the time lag involved in starting up a heating system.[(7)] He estimated the additional power, over and above the steady loss, by calculating the quantity of heat required to raise the wall temperature from cold to its steady value. He realised that this was only an approximation.

"If therefore heat is supplied only during the day, the amount supplied must be sufficient to provide for the loss that takes place at night as well as during the day. The orthodox method of calculating heat losses either leaves this essential fact out of sight or attempts in a somewhat feeble manner to provide for it by the addition of an arbitrary percentage. This addition is a proof that so far as the coefficients employed are found to be satisfactory in practice they are essentially based on the results of practical experience, and not on their correctness from the absolute or scientific standpoint... . It is not a question of scientific accuracy, but of practical adequacy."

In spite of this castigation, Barker goes on to advocate an approximate method of calculating heat losses (neglecting U-values) and adding 15% for rooms heated only during the day, or 35% for spaces which are infrequently warmed.

Little further progress, either theoretical or empirical, was made until the classic work of Nessi and Nisolle (1947).[44] In it they developed a complete theory based on the assumption of a sudden rise of indoor temperature (a so-called unit step), and defined two parameters which they termed "fonctions d'influence" — one, $g(t)$ referring to the effects of a change of inside temperature, and the other $e(t)$ the effects of a change of outdoor temperature (Fig. 13.4).

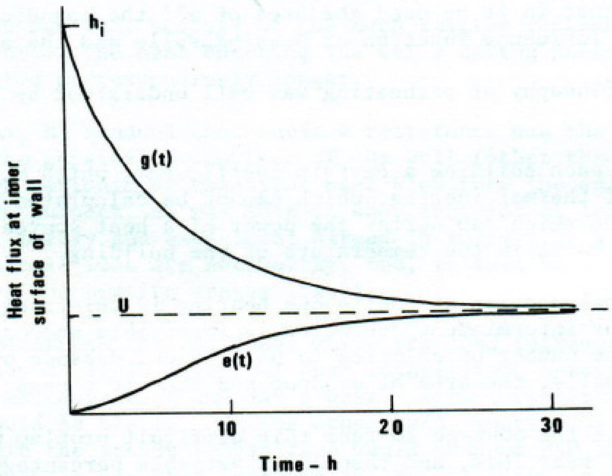

Fig. 13.4. Influence functions.

Unfortunately, the integrals involved were difficult to evaluate. Nessi and Nisolle devised a mechanical integrator with graphical output to perform the calculation of $g(t)$ and $e(t)$, and they presented the results in tabular form. Even then, the completion of the calculations for specific cases involved the summation of a series of values of $g(t)$ for the separate elements of the structure, and a second summation if the temperature variation was different from a unit step.

It is feared that Nessi and Nisolle's work was little applied in practice, even in France, though Cadiergues *et al.*[13] made some attempt to simplify its application (*ca.* 1952).

A. F. Dufton, working at BRS, made what are possibly the first scientific experimental studies of intermittent heating. He was concerned to show the value of low-thermal capacity linings (such as panelling or carpets) in intermittent heating. He developed a very simple formula to calculate the preheating time for a homogeneous wall.[21] This was extended by Griffith and Horton to apply to two-layer walls, and provided the theoretical basis for low-thermal capacity linings.[26]

The "absorbance" method by Elmer Smith (1941) was based on sound theory, but was developed to a practical design tool. It was in essence similar to that of Nessi-Nisolle, and suffered from the same disadvantages. Smith recognised the significance of internal furnishings, etc. as contributing to heat storage. Earlier a number of graphical methods, based on the Schmidt technique, were evolved for infrequently used buildings such as churches, and these methods were used by the gas industry in the United Kingdom (*ca.* 1945) for estimating the necessary heating power.

A little known work by Shklover[56] developed the matrix analysis of sinusoidal temperature waves through single and compound walls. Shklover's work is notable in that it introduces the admittance, though it is not so called. In 1949, Dusinberre's textbook on "Numerical Analysis of Heat Flow" was published.

Application of matrix analysis, and of the response factors proposed in 1956 by Brisken and Reque and developed by Stephenson[58] in Canada had to await the evolution of the computer.

Others, notably Stoef,[59] E. Harrison[28] and Barcs[5] have also made contributions both to system design and building design. Marmet made use of the electrical analogy of heat flow, and produced charts by which the impedance of a structure could be found. His theory, unlike the admittance procedure, included both amplitude and phase of the heat flow. He did not, however, apply his theory to the practical problem of designing for intermittent operation.

Krischer studied the pull-down time for refrigerated stores (the converse problem), and Bruckmayer (1951) sought to establish a simple parameter which would define heating or cooling rates. He chose a time constant Q/U which is valid only for the effects of internal changes, and which moreover is difficult to calculate precisely, because of the effects of ground storage on the value of Q. Nevertheless, it afforded some useful, if temporary, means of comparing simple and complex structures.

Experimental studies of intermittent heating were carried out by HVRA in the 1960's, and these led to a new empirical design tool (Fig. 13.5), which for the first time introduced the thermal inertia of the heating system.[11]

From its introduction, off-peak electric floor-warming led to conflict with the traditional heating industry. It seemed to the latter that the electrical designers were providing systems which were manifestly too small to give the required temperatures. This is perhaps surprising, for in 1934, Smith[57] (concerned with water-storage systems) insisted that the overall design must ensure a true heat balance, i.e. the heat put in during the off-peak charging period must equal the 24-h usage. For the design of systems to be operated intermittently, Smith, like Box, considers the heat required to raise the mean wall temperature to the desired steady-state value. He shows that for a 12-in masonry wall, this may be 32 Btu/ft^{2}h for a 3-h preheating period, as compared with the U-value of 6 Btu/ft^{2}h. He recommended the use of Rietschel's intermittent heating formula; he preferred night set-back to complete shut-off.

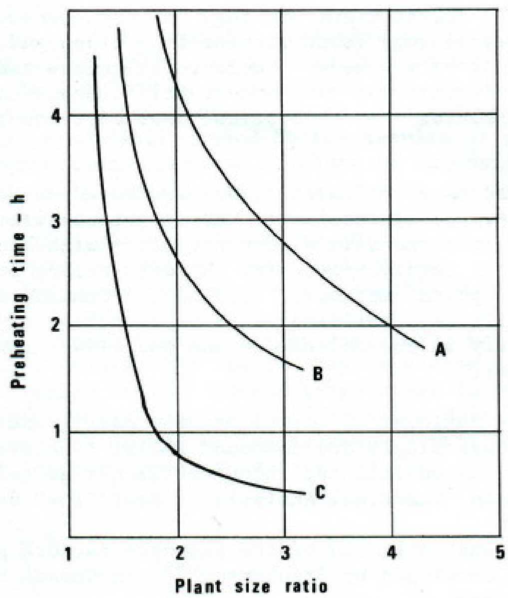

Fig. 13.5. Preheating time.

The conflict is well illustrated in a paper by H. Bruce to IHVE, and the discussion on it is sharp and pointed, if not actually acrimonious. But the investigations to which it gave rise proved valuable in the development of methods of estimating energy consumption in any kind of system. A scientific study of intermittent heating and of off-peak floor-warming was carried out by HVRA between 1958 and 1960, and this brought to light the major reasons for the apparent discrepancies — the importance of miscellaneous heat gains from other sources, the significance of the thermal capacity of the building, and perhaps excessive allowance for ventilation. All these factors tend to reduce the energy use in off-peak systems below that which the conventional theory led the engineer to expect.

At about the same time, Danter and his colleagues at BRS were developing the admittance procedure, apparently in ignorance of the work of Shklover, but the use of computer evaluation of the matrix, and the restriction to a 24-hour period, made tabulation and the application far simpler. The HVRA empirical approach, and its estimates of input power, and energy saving, were confirmed by Danter's theory.

The great advance represented by the admittance procedure is that it is applicable not only to intermittent heating, but also to problems of summer cooling. Billington and Harrington-Lynn both applied the concept to the estimation of preheating times and the corresponding energy consumptions. The final advance was the introduction by Billington of a dimensionless parameter relating admittance and transmittance to describe the thermal weight of a structure (1974-5).

In very recent years, following the energy crisis, intermittent operation has become the norm, for commerce and industry as well as for the householder. This being so, it has become essential to design for this mode of use, rather than the steady state as had previously been the case. Even more important, perhaps, is the current realisation that it is necessary to design the *thermal* properties of the structure also.

13.9 HEAT EMISSION FROM PIPES AND RADIATORS

Early attempts to determine the heat emission from pipes and radiators were inevitably crude, and often unsound in theory. The work of Tredgold, Hood and others was typical. Both Tredgold and Hood applied their figures directly as the emission from heating pipes, even when steam was used as the medium.

Dulong and Petit studied the heat loss from surfaces by radiation and convection, and proposed appropriate physical laws. Péclet used their data to estimate the emission from the pipes and pipe coils then used for heating. Box adopted a similar approach, and gave emission tables for steam and water pipes of various diameters and at several temperatures.[12] He quotes the following formulae for the loss from cylinders:

horizontal cylinders: $h_c = 0.421 + 0.307/r$ Btu/ft^{2}h F

where r is the radius in inches

vertical cylinders: $$h_c = \left(0.726 + \frac{0.2163}{\sqrt{r}}\right)\left(2.43 + \frac{5.49}{\sqrt{h}}\right) \times 0.2044$$

where h is the height in in and r the radius, also in in. He knew too that the convection from a vertical surface depended on its height, the coefficient decreasing with increasing height.

Dye, writing at the turn of the century, gave values for pipe emission identical with those of Box for single pipes. For pipe banks, 5% was to be deducted for each additional pipe. For vertical pipes, the emission was taken as 10% less than for horizontal pipes.

The heat loss from pipes has been extensively studied during the past half-century. Rietschel, and Fishenden and Saunders, carried out much experimental work, especially on heat exchangers. Prandtl and Nusselt developed the theoretical aspects, and their use of dimensionless numbers enabled vast quantities of experimental data to be correlated.

These values were, of course, not applicable to radiators or convectors. When these came to be used, in the latter part of the 19th century, it became necessary for manufacturers to state how much heat would be emitted. The first tests to determine the output of steam radiators were made by Mills and others in the 1870's, by Barrus, by Monroe (1884-96), by Baldwin (*ca.* 1888) and by Carpenter (1900-1) in America. Köerting in Germany and Ser in France gave figures appropriate to their hot water equipment.

Baldwin refined Tredgold's method.[4] The water capacity and water equivalent of the radiator were first found. The radiator was then filled with hot water and allowed to cool from 200°F to 150°F in a constant temperature room. From the data so obtained, the "heat units" lost per hour could be computed. He obtained a value of 2.01 Btu/ft^{2}h$^\circ$F. He also used a more soundly based method, in which he measured the temperature drop through the radiator and the flow rate, or the volume of condensate, in the steady state. He recommended an output of 2 Btu/ft^{2}hF as the design figure for both direct and indirect radiators.

Barrus used a free-standing steam radiator, and found the emission to be 2 Btu/ft^{2}h$^\circ$F. Monroe used the same method in his earlier tests, obtaining 2.07 Btu/ft^{2}h$^\circ$F. In his later tests, the radiator was placed against the wall of a test room, and thus in a more practical position. He found the emission to be 1.6 Btu/ft^{2}h$^\circ$F. Carpenter used a free-standing radiator in a small cubicle, and because the air circulation round the appliance was less restricted, his emission figures were higher than Monroe's later data.

Professor Carpenter showed (1900-1) that the emission from a radiator was less when superheated steam was used than with saturated steam at the same temperature. Tests by Mills, and reported by Carpenter, showed that for small radiators the emission was greater than that estimated from Péclet's radiation and convection coefficients; but for large radiators the emission was smaller, due to the mutual re-radiation between sections. Mills found emissions of the order of 2 Btu/ft^2h^oF, and it was assumed to vary linearly with temperature difference. Carpenter recommended lower values — 1.8 for a temperature difference of 150^oF and 1.7 for a difference of 110^oF between radiator and air.

Writing in 1902, Monroe[42] commented that the practice of rating radiators in terms of their surface area was an arbitrary one, which had been adopted for want of a better. He himself measured the area by carefully covering the surface with pieces of paper.

He recognised that only part of the surface could radiate to the enclosure. For a single-column radiator, only about 80% of the surface could "see" the room and radiate to it; for 2- and 3-column radiators, the corresponding figures were 45-55% and 35-45%. The implication is that the emission per ft^2 of a multi-column radiator is less than that of a single-column one, and moreover the ratio of convection to radiation rises. Monroe deduced that since, for a single pipe in still air, about half the emission is by radiation, the relative emissions for radiators would be:

pipe	100%	50% convection	50% radiation
1-col radiator	90%*	56% convection	44% radiation
2-col radiator	75%*	67% convection	33% radiation
3-col radiator	70%*	71% convection	29% radiation

*taking pipe as 100%.

Rietschel embarked on an extensive programme of testing at Charlottenburg in 1896, and this work was continued by Brabbée between 1917 and 1927. Rietschel's tests were probably the first scientific and comprehensive trials on a wide variety of radiator patterns, and on tube banks, both plain and finned. The appliance was placed in a test room 6.5 x 4.5 x 4 m, itself within a large laboratory. It is not clear from his book how, or if, the test room temperature was controlled. The output from a hot water radiator was determined in the steady state from the weight of water passing and its temperature drop. For steam, the weight of condensate was measured. Rietschel studied the effect of fluid velocity, of air speed over the surface, of the spacing between the panels of a multi-panel radiator, and of the number of sections in a sectional radiator. His data were used by engineers throughout Europe for many years.

Barker quoted extensively from Rietschel's data. He believed, however, that the results, obtained some years before Barker's book appeared in 1912, were probably 15 to 20% low compared with more recent tests. For 2-column radiators, with sections at 3-in centres, the emission was taken by Barker to be 1.5 Btu/ft^2h^oF, though this value was known to depend on the temperature. The total emission, from Rietschel's data, appeared to be proportional to the temperature difference between the radiator and the air. Rietschel expressed his results in terms of the mid-temperature (i.e. mean of flow and return) though Barker criticised this as not being the true mean temperature of the surface.

The reduction in output due to shelving, casing or shielding of the radiator was known, and estimated at a maximum of 20%.[23]

The National Radiator Company, makers of the "Ideal" range of equipment, gave what was the most complete and accurate data on heat output available during the first half of the present century. The "Ideal" manual (though in essence a manufacturer's catalogue) was commonly used as a design guide in America and Britain, and perhaps in other European countries also. The Manual of 1930 quotes the results of experiments to determine the effect of painting radiators;[43] ordinary paint had no effect, but metallic paint reduced the radiation component of column radiators by some 45%, and the total output by 12% (a figure which had been found much earlier by Monroe). Varnishing over the metal paint restored the emission to the original value (because the emissivity of all non-metallic paints is close to 0.9).

Determinations of the heat output of "indirect" radiators were made by Richards, Baldwin, Mills and others in the period 1873-1885. Monroe[42] correlated the data on two appliances — the Gold pin radiator and the Whittier indirect radiator — plotting the total output (H) against the air flow (V) at constant steam and air inlet temperatures. He found:

$$H = a\,V^n$$

where n had the value 0.79 for the Gold and 0.68 for the Whittier radiator. He goes on to regret the lack of recent data on more products (this was written in 1902).

Modern radiator tests are carried out under closely specified conditions in either a warm-wall booth or a controlled-temperature cold-wall room. Neither exactly represents the practical situation. In the 1930's, the practice arose (in America) of adding a percentage (usually 15%) to the test output, the final figure supposedly being the output which would be obtained in real rooms. Coles[16] states that the origin of this addition for "heating effect" lies in the different vertical temperature gradients produced by convectors and radiators, so that different outputs were required to give a specified air temperature in the test room at 30 in above the floor (Fig. 13.6). These additions were codified in the USA in 1947 for convectors and in 1950 for baseboard heating. Similar additions were applied in Belgium, but they were deprecated in Britain.

Heat-Distributing Equipment

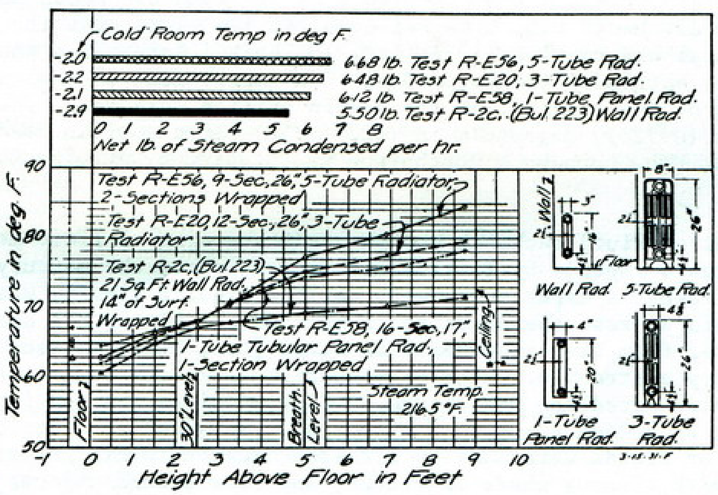

Fig. 13.6. Room-temperature gradients and steam-condensing rates for four types of cast-iron radiators with a common 30-in level temperature.

It is not known when it was first realised that the emission from radiators and convectors was not in fact proportional to the temperature difference between the medium and air. Dulong and Petit's and Péclet's work must have suggested this; yet all experimenters from Tredgold to Carpenter assumed a linear dependence on temperature difference. In America, of course, where steam heating was widely used, it was sufficient to quote emissions for temperatures around 212°F. For hot water radiators, Rietschel quoted values of K varying with the temperature, and Barker adopted the same procedure. Neither proposed the current 1.33 power law for radiators, nor the 1.25 power law for convectors. Barker and Kinoshita,[34] as a result of some 200 tests on radiator emission, showed that the expression

$$\text{Output} = k.(\Delta t)^{1.3}$$

applied to both steam and hot water radiators. Additionally, they showed that a shelf above a radiator reduced the output, by interfering with natural convection, that the emission was independent of the water flow rate, and that the (top, bottom, opposite ends) connections gave a 12% greater output than when connection at both ends were at the bottom.

Hoffman and Raber knew, in 1913, that radiator output depended on its height (increasing height leading to lower output per unit area, since the upper part is washed by warm air rising from the lower part). They found:[29]

Standard height	30 in	1.7 Btu/ft^{2}h°F
	16 in	+10%
	48 in	-10%

The aspect ratio also affects the temperature gradient and uniformity, and hence the output. A long low radiator gives greater uniformity and a lower gradient — a fact which was known in 1899.[35]

13.10 FLUID FLOW

Leonardo da Vinci (1452-1519) gave the first recorded sketches of the form of a liquid jet issuing from an orifice; he also proposed a design for an anemometer. Newton (1642-1727) measured the dimensions of the *vena contracta* and introduced the coefficient of discharge (which he set equal to $1/\sqrt{2}$) to make the theory accord with experimental observations.[54] In 1718, Polavi determined the coefficient experimentally, obtaining the value 0.62 — an improvement on Newton's value. Later determinations of the coefficient for a thin orifice were made, *inter alia*, by Girard in 1821 (0.725), Lagerhelm in 1822 (0.58), Aubuisson in 1826 (0.65) and Péclet (0.65). Flow through a mouthpiece was studied by Aubuisson, Eytelwein and Péclet.

The development of fluid machines and the understanding of fluid mechanics was given considerable impetus by Torricelli's invention of the mercury barometer (1644) and by Pascal's experiments with various liquids (1647), when he concluded that "in a fluid at rest the pressure is exerted equally in all directions". In 1686, Marriotte, from his experiments on water jets, first appreciated that the force exerted by a stream of water is proportional to the square of the velocity of flow. He also noted the resistance to flow in pipes and the increased resistance due to sudden changes of direction. Substantial contributions to the understanding of fluids were made by Sir Isaac Newton in his *Principia Mathematica*. Newton dealt with viscous shear in fluids, and flow around objects immersed in a moving fluid.

The pitot tube was invented by Pitot in 1732: he used two separate tubes, one being bent at 90°, to give the facing and static pressures in a fluid stream. In 1884, the Prussian Mining Commission investigated various methods of measuring air speed. The accuracy of the pitot tube was verified; and they found too that a thin plate orifice could be used, the volume being given by:

$$Q = C\,A\,\sqrt{gH}$$

where H = pressure drop (ft of air)
A = area of orifice
C = discharge coefficient, = 0.64 for a round orifice
= 0.61 for a square orifice.

In the same year that the pitot tube was invented, Couplet made a study of the flow of water in the pipe system serving the fountains at Versailles. Martin(41) regards these as the first useful experimental data; from them, Couplet concluded that the loss of head was proportional to the square of the velocity.

Antoine Chézy experimented on water flow in pipes over a period of some years, and in 1775 gave the first rational statement on pipe friction. His expression for turbulent flow was:

$$v = C\,\sqrt{m_l\,i}$$

where v = fluid velocity
m_l = hydraulic mean depth
i = hydraulic gradient, head loss per unit length.

The expression was modified by Prony in 1794, who suggested that the resistance depended on both the first and second powers of the velocity.(54) Neither Chézy nor Prony took any account of the surface characteristics of the pipe.

In 1738, Daniel Bernoulli published his *Hydrodynamica*, containing the broad outline of the now famous Bernoulli equation. The principles were later used by the Italian physicist Venturi in 1797 in the construction of the flow meter which bears his name.

Rather earlier, in 1766, Borda determined the energy loss which occurs at a sudden contraction or other change of section. He seems to have been the first to include the factor $2g$ explicitly in a flow equation.

The law for laminar flow in a tube was deduced by Poiseuille (1841) from experimental work; the expression was derived theoretically by Neumann and Hagenbach in 1858-60.

Aubuisson put forward in 1834 some principles of fluid flow, which led to a formula of the type:

$$H = a\,L \cdot \frac{v^2}{g m_1}$$

In 1854 Hagen gave, for the resistance in turbulent flow:

$$H = \rho\,\frac{L\,v^{1.75}}{D^{1.25}}$$

thus predating some of Osborne Reynolds' discoveries. Hagen, considering his own experiments and those of earlier investigators, concluded that frictional losses could be simply represented by using fractional indices. However, some years earlier (1845) Weisbach had proposed an expression of the form:

$$h = 4\zeta \cdot \frac{L}{D} \cdot \frac{v^2}{2g}$$

with the friction coefficient ζ varying with $1/\sqrt{v}$, thus allowing frictional losses to be expressed in terms of velocity head. This was akin to Girard's formula for the flow of gases, and to Aubuisson's.

This form of expression is perhaps more usually associated with D'Arcy (and in fact often bears his name), though Rouse and Ince state that D'Arcy's formula, published in 1857, included terms in both v and v^2 as well as a dimensional coefficient involving roughness. The ASHRAE Guide attributes the expression to Aubuisson.

Experiments by Weisbach, Ledoux, Rietschel, Unwin and others showed that the friction factor ζ varied with both velocity v and pipe diameter d.

Péclet, about 1860, carried out an extensive series of tests to determine the resistance of various pipe fittings (i.e. bends, valves, tees etc.) to the flow of water. Rietschel(52) expressed his own results in terms of velocity head loss ($v^2/2g$):

sharp elbow	1.0 velocity head
round elbow	0.5
return bend	0.8
sudden enlargement	1.0
open cock	0.1-0.3
open valve, ordinary seat	0.5-1.0

Although the second half of the 19th century was the age of steam power, remarkable advances were still made in the understanding of fluid flow and the development of fluid machinery. Stokes produced the law which bears his name and relates to the rate of fall of a solid sphere in a viscous fluid. Horace Lamb published his classic textbook on *Hydrodynamics* in 1879. But the outstanding investigator of this period was Osborne Reynolds.

In 1883, Reynolds discovered the two modes of motion in fluids known as "streamline" and "turbulent", and he went on to explain how the transition takes place at a "critical velocity". He was the first to show a definite relationship to exist between the frictional loss due to water flowing in a pipe and certain physical factors which can be expressed in the form of the dimensionless number vd/ν, the Reynolds Number.

His work enabled some of the apparent discrepancies in the earlier work of Poiseulle (1846) and Darch (1857) to be explained. It was around this period that William Froude and later his son Robert, carried out their experiments on the resistance of surfaces of different shapes and finishes being drawn through the water at different speeds.

For the flow of air, Girard (1821) demonstrated that:

$$v^2 = \frac{2g}{k}\frac{D\rho}{l}$$

where k is a coefficient of friction. Péclet doubted Girard's suggestion that k depended on the material: he believed it to be a constant (0.024) for a range of pressures, pipe diameters and length, for both air and coal gas. Péclet's book[46] contains a very detailed account of his experiments on the flow of gases through orifices, changes of section, a succession of bends and so on. The trials, which were carefully executed with good accuracy, confirmed and extended the work of Aubuisson and of Eytelwein.

Apart from the relatively small quantity of data acquired prior to 1850, our knowledge of the friction of air in pipes and ducts may be said to date from Weisbach's experiments. The theory had been established by Montgolfier and Bernoulli. Box quoted the pressure loss for air flowing through a pipe as:

$$H = \frac{v^2 l}{(3.7d)^5}$$

where H = pressure loss, in w.g.
v = volume, ft^3/min
l = length of pipe, yards
d = pipe diameter, in.

The implicit assumption is that the friction factor is constant in all circumstances.

Weisbach's contribution was that of the experimental determination of the friction loss. His book on *Fluid Mechanics* was published in Germany in 1855. The pressure drop in a circular duct was:

$$H = 4\zeta \cdot \frac{l\ v^2}{2g\ d}$$

as for water in pipes, and he found that the value of 4ζ ranged from 0.015 to 0.026 for straight pipes of different kinds. A similar value was found by Ledoux. For 90° elbows, Weisbach found 4ζ = 1.41 to 1.61; and for long 90° bends, 4ζ = 0.47. Unwin found the coefficient 4ζ to vary with the diameter and the roughness. It will be noticed that there is as yet no realisation of the fact that 4ζ depends on the velocity or the Reynolds Number.

The Prussian Mining Commission found the resistance to air flow in cast iron pipe to be proportional to:

$$d^{-1.37} v^2 \ \rho^{2/3}$$

while Rietschel's tests at Charlottenburg yielded:

$$R = 0.058\ v^{1.85}\ d^{-1.26}$$

where R = resistance, in wg/ft run
v = velocity, ft/s
d = diameter, in

and Fritzsche's work in 1907 gave:

$$R = \frac{b\ \rho^{0.852}\ v^{1.924}}{d^{1.281}} \quad (\rho \text{ in } lb/ft^3)$$

These expressions represent a return to the earlier empirical form.

Prandtl and his students were responsible for important advances in the theory of fluid flow and its experimental verification. Blasius (1908) published an analytical solution which put Prandtl's qualitative theory into quantitative terms; and this in turn was fully verified by later experiment. In his 1911 paper, on similarity laws in fluid flow, Blasius showed that the resistance coefficient for smooth pipes was a unique function of the quantity vd/ν (the Reynolds number) - $f = 0.0791\ Re^{-0.25}$. A subsequent paper, in 1913, contained a correlation plot based on data from Saph and Schoder at Cornell as well as Blasius' own on water and air.[54] A year later, Stanton and Pannell at the National Physical Laboratory extended the correlation to include data on oil.

Later work by Lees (1915), Nikuradse (1932), Colebrook and White (1937) and Moody (1944), and theoretical work by Prandtl and Karman (1930-2) has enabled f to be determined for smooth, commercial and fully rough pipes. For smooth pipes, the friction coefficient is related to the flow conditions by the expression:

$$\sqrt{1/f} = 4\log_{10}\left(\frac{Re\cdot\sqrt{f}}{1.255}\right)$$

while for fully rough pipes it is given by:

$$\sqrt{1/f} = 4\log_{10}\left(\frac{3.7d}{k}\right)$$

where k is the absolute roughness. The latter coefficient is thus independent of Reynolds number. The transition from smooth to fully rough pipes was studied by Colebrook and White, who proposed the equation:

$$\sqrt{1/f} = 4\log_{10}\left(\frac{1.255}{Re\cdot\sqrt{f}} + \frac{k}{3.7d}\right)$$

to cover the range of commercial pipes. These equations form the basis of most current fluid flow tables.

13.11 PIPE AND DUCT SIZING

As early as 1824, Tredgold used the basic formula:

$$h = v^2/2g$$

to compute gas flow in chimneys and to size steam and gas mains. He did not, however, consider the effect of surface roughness. Heigelin refers to Tredgold as being the first to give scientifically based calculations of fluid flow.

Hood's book[30] on warm-water heating, published in 1844, was an epoch-making volume. He calculated pipe friction from Prony's formula, though he still complained that "a simple and correct formula on this subject is still a desideratum".

The desirability of sizing the main flow pipe in relation to the heat required was known and remarked. The area of this flow pipe was required to be less than the total area of the branches, in order to increase the velocity in it; and this arrangement was said to have the advantage of the smallest heat loss.