XIV.—*An Account of some Experiments on the Expansion of Water by Heat.* *By the late* T. TREDGOLD, *M.Inst.C.E.*

THE expansion of water, by increase of temperature, is one of those experimental subjects that has not received the degree of attention its importance would lead us to expect; but, as even the smallest addition to any part of knowledge contributes towards its increase, I have ventured to send this mite for the consideration of the members of the Institution.

I began by a series of trials with a thermometer, containing water instead of mercury, to find the point at which the volume of water is a minimum, by cooling successively down to 32° with snow and water, and observing the decrease of bulk, which continued till the temperature was 40°; the rise again was then sensible. In like manner by cooling, the decrease continued till the temperature was about 39°, when the rise became sensible. So small and uncertain, however, was the rate of increase or decrease, that we may practically estimate 40° as the temperature corresponding to the maximum density of water.

Having marked the tube at the point when the temperature was 40°, and also another point within the range of the tube, I divided the distance between these, into four equal parts. With this precaution I immersed the water thermometer, and a mercurial one, in a vessel of hot water, and as it cooled compared the temperatures as the water contracted to each division on the tube. The mean of several trials was as follows:

Temp.	112°	4th or upper division.
—	104°	3d.
—	90°	2d.
—	74°	1st division.
—	40°	maximum density.

I intended to repeat the trials and to correct these numbers; but the cold weather commenced, and instead of attending to the higher degrees of heat, my attention was directed to the lower ones. The bulb of the thermometer was immersed in a mixture of snow and salt, and a mercurial one placed beside it, but I found the two were not alike affected by the mixture; the water thermometer rose rapidly till it arrived at, or very near to the third division on the tube, when it exploded. At the moment of explosion, the central part of the mass of water, and that in the tube were both perfectly fluid, and the fragments of the bulb were lined with a thin coat of ice, beautifully crystallized. The fractured bulb presented a singular appearance, the whole being cracked into very fine gores, somewhat less than one-twentieth of an inch in breadth at the middle, and exceedingly regular.

The temperature of a mixture of snow and salt is $-5°$, or 5 degrees below zero; hence, if the expansion below 40° had been the same as far above 40° the thermometer ought not to have risen quite to the second division; but, as it rose very nearly to the third division, it seems that the expansion below 40° is much greater than at a corresponding number of degrees above 40°; and that the common opinion is not quite correct in this respect.

I have not had leisure to follow up these trials, for they consume an immense quantity of time; but from those made by others, and checked by my own, I have deduced a formula for calculating the expansion at any temperature.

If we consider the force with which matter resists the entrance of heat to be inversely as the square of the distance of its elementary atoms; then, the bulk being as the cube of the distance, the resistance to heat will be inversely as the square of the cube root of the volume, and the increments of expansion by heat directly as the $\frac{2}{3}$ power of the volume. The sum of the increments will, therefore, be as the $\frac{5}{3}$ power of the volume, and the equation must give zero at 40°; hence it will be $A\ (t-40°)^{\frac{5}{3}}$ = the expansion, where A is a coefficient to be found by experiment, and t denotes the temperature.

The calculation is easy enough by logarithms, for, $\log A + \frac{5}{3} \log\ (t-40)$ = log of the expansion; or $3\left(\frac{\log \text{expansion} - \log A}{5}\right) = \log\ (t-40°)$.

The formula in the last form applies to my experiment, and becomes $3\left(\frac{\log \text{expansion} + 3{\cdot}09555}{5}\right)$ = the $\log\ (t-40)$, the expansion at 112° being considered unity; hence the comparison is easy, and is as under.

Expansion	Temperature by experiment.	Temperature by formula.
1	112°	112°
0·75	104°	100°
0·5	90°	87°
0·25	74°	71°
0·00	40°	40°

The coincidence is as near as we could expect, considering how difficult it is to insure perfect accuracy in the observations; but, before we proceed further in experiment, it is natural to ask how it will agree with others already made.

The expansion of water from 40° to 212° has been found to be 0·04333, its bulk at 40° being unity. By substituting this value in the formula, we find the coefficient A, and have the rule $\frac{5}{3}\log(t-40)+(-6{\cdot}910909)$, or its equivalent $\frac{5}{3}\log(t-40)-5{\cdot}089091$ = the log of the expansion.

The formula being in this case derived from a probable hypothesis, it is more likely to express the true expansion, than one made out merely to fit a short range of experiments. The absurd conclusions which may follow from an experimental rule are avoided; and that such conclusions do arise out of formulæ made to fit a particular set of experiments, we have an evidence in the case under consideration; for Dr. Young* has given a formula for calculating the expansion of water, which becomes negative when the temperature is 540°; indicating that water would decrease in bulk, by increasing its temperature above that point; this is a circumstance too improbable to guide us in any practical application of the formula.

The annexed Table shows the bulk and expansion for a few temperatures.

Temperature.	The expansion.		Bulk by formula.	Temperature.	Expansion by formula.	Bulk.
	By experiment.	By formula.				
40°	0	0	1·0000	400°	0·1484	1·1484
64°	0·00133	0·00162	1·00159	800°	0·5155	1·5155
102°	0·00760	0·00791	1·00791	1000°	0·7610	1·7610
212°	0·04333	0·04333	1·04333	1171°	1·0000	2·0000

In my own experiments, the formula was in defect in the temperatures between 40° and 112°; here it is in excess; the difference may arise from the expansion of the glass in my trials. According to this formula, water will expand to double its bulk at 40° by a temperature of 1171 degrees. What would be the force of the steam to confine it to the liquid state at that temperature? There is abundant scope for curious research in this matter: it is one where speculative opinion feels the want of experience.

I am not aware of there being any experiments on the expansion of water above the boiling point. When I find an opportunity, I intend continuing the

* Lectures on Natural Philosophy, Vol. II. p. 392.

series as I can, using something to colour the distilled water, for facility of observing; and I trust soon to be able to communicate some account of my progress *.

* It is not certainly known whether Mr. Tredgold ever followed out the consideration of this interesting subject; but, as he made no further communication thereon to the Institution, and his premature death took place soon after the date of this paper, it seems probable that his experiments were never resumed.